come solutions of (1). Then even solutions are denoted by $Mc_r^{(1)}(x, q)$, and the odd ones by $Ms_r^{(1)}(x, q)$. For $q = 0$ the eigenvalues of (2) are double eigenvalues. Then $Mc_r^{(1)}(x, o)$ and $Ms_r^{(1)}(x, o)$ become linearly independent solutions of the same differential equation. They are in this case hyperbolic cosines and sines respectively.

Volume 1 was devoted to the functions mentioned above and their derivatives. Volume 2 extends the range of values tabulated for these functions, and also tabulates a second linearly independent solution and its derivatives. As in Volume 1, the functions are not tabulated directly. Thus, we have, for example, $Mc_r^{(2)}(x, q) = e^{-rx}Tc_r(x, q)/rM_r(q)$.

The extraction of the factor e^{-rx} leads to a function $Tc_r(x, q)$. These functions are readily interpolable in both x and q. Necessarily, therefore, this table must be used in conjunction with a table of exponential functions. Tabulated data for these functions are provided for $x = 0(0.02)1$; $q = 0(0.05)1$; $r = 0(1)7$; 7D, and $x = 0(0.01)1$; $q = 0(0.05)1$; $r = 8(1)15$; 7D.

An auxiliary set of tables is provided for a second set of linearly independent solutions and their derivatives denoted by $Dc_r(x, q)$, $Ds_r(x, q)$, $Ec_r(x, q)$, $Es_r(x, q)$.

Tables V–XII provide data by means of which all solutions can be computed for $x = 1(0.02)2$, $\sqrt{q} = 0.5(0.02)1$, to 7D. These are tabulated in terms of a class of functions denoted by $Fc_r^{(j)}(x, q)$, $Fs_r^{(j)}(x, q)$, $Gc_r^{(j)}(x, q)$, and $Gs_r^{(j)}(x, q)$, where $j = 1, 2$. These again are so defined as to lead to smooth data but must be used in conjunction with a table of Bessel functions.

These tables in conjunction with the asymptotic formulas provided in the introduction to Volume 2, provide a thorough numerical knowledge of the solutions of equation (1).

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24 [L].—ROBERT SPIRA, *Check Values, Zeros and FORTRAN Programs for the Riemann Zeta Function and its First Three Derivatives*, University Computing Center, Report No. 1, University of Tennessee, Knoxville, Tennessee. Copy deposited in the UMT file.

This report contains documented FORTRAN programs for calculating $\zeta^{(k)}(s)$, $k = 0(1)3$, and also the following tables:

(a) $\zeta(.5 + it)$, $|Z(t)|$: $t = 500, 1000, 2000$; 18D.
(b) $\zeta^{(k)}(\sigma + it)$: $k = 1(1)3$: $\sigma = -2(1)0(.5)1(1)3$,
$t = 0, 10, 50, 100, 200$ (except for pole);
also $\sigma = 0(.5)1$, $t = 500, 1000$;
Accuracy: Nearly all 10D or 10S.
(c) Zeros of $\zeta(s)$: First 30, 13D.
Zeros of $\zeta'(s)$, $\zeta''(s)$: $-1 \leqq \sigma, 0 < t \leqq 100$, 10D.
(d) $-b_\mu = B_{2\mu+2}/B_{2\mu}$: $\mu = 1(1)50$, 30D.

Cross checking was accomplished by also computing the derivatives by a differences program that is included. The zeros of $\zeta'(s)$ and $\zeta''(s)$ were obtained from previously known approximations by a Newton's method program (also included).

AUTHOR'S SUMMARY